The freestream velocity u is determined as a function of gas temperature by measuring the total (static plus dynamic) and static pressure of the stream. At low values of Mach number, Bernoulli's equation can be used and gives

$$u = \{2(R/M)T[(P_t/P) - 1]\}^{0.5} \tag{10}$$

For supersonic streams, u again can be determined, as a function of gas temperature, from a knowledge of the freestream static and total pressure by using the compressible flow relation for constant specific heat:

$$u = \left\{\frac{2\gamma}{\gamma - 1}\left(\frac{R}{M}\right) T \left[\left(\frac{P_t}{P}\right)^{(\gamma-1)/\gamma} - 1\right]\right\}^{0.5} \tag{11}$$

The values for the transport properties of each constituent of the jet can be obtained from Ref. 4 and combined to calculate the mixture properties by using the nomographs of Ref. 5. The specific heat for each component also can be obtained from Ref. 4. This procedure was used herein to measure the temperature of the combustion products of the cyanogen-oxygen flame. This is a low subsonic jet ($M <$ 0.2) having an adiabatic flame temperature of about 8700°R but found from spectrographic measurements to be actually about 8000°R. (For a description of the burner and associated apparatus, see Ref. 6.)

Since the composition of the combustion products of the cyanogen-oxygen reaction, carbon monoxide and nitrogen, remains relatively constant with change in temperature,[6] that is, there is little dissociation, and since the radiative emissivities of these gases are very low in the temperature range under consideration, the primary heat transfer mode is one of convection. Therefore, only the heat transfer coefficient calculated from the boundary layer equations is required to describe fully the overall mechanism of heat transfer. A schematic diagram of the probe used for the temperature determination is shown in Fig. 1. In order to reduce heat losses, 1) the thickness of the front face l is very small compared to the diameter, and 2) the probe is evacuated. A flat-faced probe was chosen, as this configuration is relatively easy to fabricate while allowing the thickness l to be machined to a very high degree of uniformity. A thermocouple welded to the stagnation point of the probe measures the wall temperature, and, by recording this with an oscillograph, the time rate of change of this temperature ($\partial T_W/\partial \tau$) is determined. The temperature of the combustion products determined by this method was found to be about 7800°R, which was within 3% of microwave attenuation and spectrographic determinations. Thus, this technique has proved to be an accurate and relatively simple means to measure the temperature of high-temperature gas streams.

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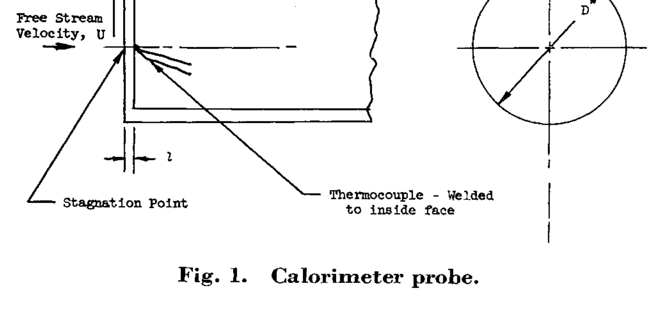

Fig. 1. Calorimeter probe.

On the Stability of a Class of Discontinuous Attitude Control Systems

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An examination is made of the stability, under external torque, of attitude control systems having fixed minimum-impulse bits delivered at unsymmetrical vertical switch lines. A difference equation is obtained for successive intersections of a single switch line. The Liapunov method is used to show that any steady external torque causes instability.

Nomenclature

c = $L_c/(L_c + L_e)$
e = attitude error
$\Delta\dot{e}$ = change in error rate due to a fixed control impulse bit
I = moment of inertia
L_c = control torque
L_e = external torque
x = dimensionless perturbation attitude error rate

A PARTICULAR class of discontinuous attitude control systems has been examined for space vehicle applications by several authors.[1,2] These systems are characterized by fixed-impulse bits delivered at unsymmetrical vertical switch lines in the error, error-rate phase plane. Figure 1 shows the limit-cycle behavior achieved by such systems under special conditions (no external torque, plant having inertia only), after a convergence period that differs depending upon the system details. It is the purpose of this note to point out that any steady external torque causes these systems to become unstable. That is, the trajectory eventually "escapes" from the inner switch lines.

Two types of trajectory may exist under external torque:

1) The control-torque-off trajectory following a fixed control impulse either intersects the same switch line or neither switch line. In the latter event, the trajectory "escapes," and a limit cycle is possible only if additional symmetrical or unsymmetrical switch lines are provided at larger (absolute) values of error. But then the behavior is unstable according to the criterion adopted and need not be considered further.

2) The control-torque-off trajectory following a fixed control impulse intersects the opposite switch line.

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A necessary and sufficient condition for stability in the first case is that the initial switch line intersection occurs at an attitude error rate equal to one-half of the rate increment corresponding to the fixed control impulse. This means that steady external torque will cause instability, since that exact switch line intersection will occur only as the result of a coincidence. The second case, in which both switch lines are intersected, also leads to instability.

A demonstration of instability for the first case follows. The well-known phase-plane equation for the system is

$$e = e(0) - [I/2(L_c + L_e)][\dot{e}^2 - \dot{e}(0)^2] \tag{1}$$

Another equation governs the change in error rate due to the fixed control impulse bit:

$$\dot{e} = \dot{e}(0) + \Delta\dot{e} \tag{2}$$

Consider a fixed-impulse control-torque-on trajectory originating at the right-hand switch line, followed by a control-torque-off trajectory that ends at the same switch line. By appropriate matching of final and initial values, Eqs. (1) and (2) can be combined to yield the following difference equation for successive values of error rate at switching:

$$\dot{e}_{k+1}{}^2 = \dot{e}_k{}^2 + c\Delta\dot{e}(2\dot{e}_k + \Delta\dot{e}) \tag{3}$$

A stable limit cycle is obtained when $\dot{e}_k = -\Delta\dot{e}/2$. This condition, which specifies that the error rate at switching is one-half of the change in error rate due to a fixed control impulse, yields the stable limit cycle solution

$$\dot{e}_{k+1} = \dot{e}_k \tag{4}$$

A dimensionless perturbation form of Eq. (3) is obtained with Eq. (4) as the reference motion by the following substitution:

$$x_k = \dot{e}_k/(\Delta\dot{e}/2) + 1 \tag{5}$$

Equation (3) becomes

$$x_{k+1}{}^2 - 2x_{k+1} = x_k{}^2 + x_k(4c - 2) \tag{6}$$

Equation (6) is a nonlinear difference equation whose solution is not obvious. Adopting the Liapunov method, a positive definite function $V(x_k)$ is found whose first difference ΔV is positive over the region of interest but zero at the origin, proving instability. The positive definite function and its first difference are

$$V(x_k) = x_k{}^2/2 \tag{7}$$

$$\Delta V(x_{k+1}, x_k) = x_{k+1} + x_k(2c - 1) \tag{8}$$

$$\Delta V(x_k) = 1 - [(1 - x_k)^2 + 4cx_k]^{1/2} + x_k(2c - 1) \tag{9}$$

The minus sign before the bracket corresponds to the physical fact that the switch line is unsymmetrical. Figure 2 shows that ΔV is positive everywhere in the region of interest ($x_k < 1.0$, $c > 1.0$), except at $x_k = 0$, proving instability.[3]

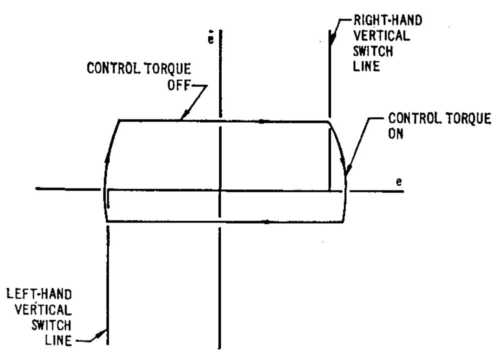

Fig. 1 A limit cycle for unsymmetrical vertical switch line fixed-impulse-bit attitude control system.

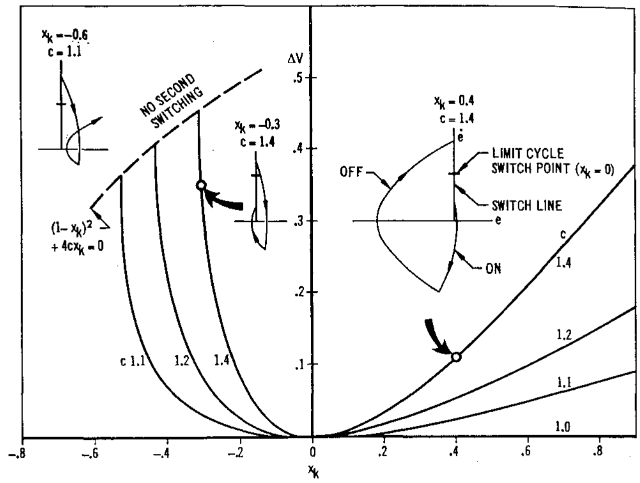

Fig. 2 Variation of the difference function ΔV with x_k and c.

The left-hand sketch of Fig. 2 is representative of trajectories having x_k and c values to the left of the dotted line labeled "no second switching." The trajectories following the initial switching at x_k do not intersect the switch line again but "escape" to the right. The other sketches illustrate two terms of the divergent series implied by $\Delta V > 0$.

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Correlation of Rocket Nozzle Gas Injection Data

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WALKER, Stone, and Shandor[1] report the results of an extensive experimental investigation of the effects of injectant molecular weight and injectant nozzle diameter on the side force induced in a rocket nozzle by secondary injection. Reference 2 contains an analysis of this problem that predicts the effect of injectant molecular weight and concludes that the interaction force (the side force minus the force arising directly from the injection momentum) is independent of injectant pressure over a wide range of pressure and, hence, independent of injectant nozzle diameter. It is the purpose of this note to compare these results.

In the experiment described in Ref. 1, the side force was measured by force transducers in a conical rocket nozzle in which the propellant was decomposed H_2O_2, and several gases (CO_2, N_2, Ar, He, and H_2) at ambient temperature were injec-

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